A REVIEW ON DIRECTIONAL FILTER BANK FOR THEIR APPLICATION IN BLOOD VESSEL DETECTION IN IMAGES

Pritee Jinturkar and Prof. A. D. Tete

Department of Electronics Engineering, G. H. Raisoni College of Engineering, Nagpur, India

ABSTRACT:

Vessel enhancement in contrast CT, MRI images is an important preprocessing step for clinical diagnosis and further processing. Hessian based methods suffer the drawbacks intensity inhomogeneity in vessel junction regions. In this paper, aim is to review a Directional Filter banks based algorithm. This algorithm decomposes an image into several directional images, reducing noise sensitivity. Each of these directional images are enhanced separately and these separately enhanced images are recombined to generate final output image with enhanced features.

Keywords: Diamond filter, Directional filter bank, Vessel detection, Frangi vessel model

[1] INTRODUCTION

Accurate analysis of the human vascular structure is important requirement for clinical diagnosis and further processing. More anatomical details help to calculate an accurate approximation of vasculatures to a higher extent. Vasculature structures for liver, heart, brain are very complex, thus more details, accuracy will help to lead to diagnose correctly as well as surgical planning, outcome assessment, and monitoring of the progression of vascular related diseases. Hessian Matrix analysis based methods are popular for the enhancement of vessels because of their ability to obtain directional and dimensional information of vessels [1]. The most commonly used Vesselness measures are the ones proposed by Sato et al. and Frangi et al. are hessian based methods. Drawbacks of these methods are that they are highly sensitive to the noise because of second order derivatives. Filter functions proposed by Frangi et al. has problems at junctions, intensity inhomogeneity inside vessels because of assumption of Gaussian profile of vessel cross-section and localized characteristics [1], while the filter functions developed by Sato et al. and Erdt et al. have problems with nearby vessels. Nearby vessels tend to diffuse into one another [2]. One of the important advantages of these approaches in this category is that vessels in a wide range of diameters and scales can be captured because of use of the multi-scale analysis [9].
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In this paper, the aim is to review the directional filter bank called (DFB), as DFB has many applications like coding, multiresolution, noise reduction etc. in image processing. Taking the advantage of noise reduction of this method, its application for medical image processing for vessel detection is developed.

The input image is decomposed by directional filter bank into a set of images called directional images, which extract the line like features in a narrow directional range. Then enhancement filters are applied for each directional image for the enhancement of the vessels. Finally, this resulted vessel enhanced images are then recombined to form a final resulting image.

The DFB is efficiently implemented via an l-level tree-structured decomposition that leads to $2^l$ subbands with wedge-shaped frequency partitioning. Figure 1 shows frequency partitioning for the $l=3$ level DFB, also DFB is a non-redundant transform, and offers perfect reconstruction, i.e., the original signal can be exactly reconstructed.

[2] Diamond Shaped Filters

Consider the 2D two-channel filter bank in Fig. 2, where the decimation matrix $M$ is a 2 x 2 integer matrix with $\det(M) = 2$. For the diamond-shaped filter bank, decimation matrix is usually the quincunx matrix $Q$ as given below. More details about the Multi-Dimensional Multirate filter banks can be found in [6][10].

$$Q = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

In the filter bank shown in Fig.2, the lowpass filter and highpass filter in analysis section are $H_0(w)$ and $H_1(w)$ respectively and the lowpass filter and highpass filter in analysis section are if $F_0(w)$ and $F_1(w)$ respectively. So the design of the diamond filter bank is reduced to the design of 2D filters $H_0(w)$ and $F_0(w)$, where $\pi = (\pi_1, \pi_2)^T$, $w = (w_1, w_2)^T$, $z = (z_1, z_2)^T$.

$$\tilde{X}(w) = \frac{1}{2} [H_0(w)F_0(w) + H_1(w)F_1(w)]X(w)$$

$$+ \frac{1}{2} [H_0(w + \pi)F_0(w) + H_1(w + \pi)F_1(w)]X(w + \pi)$$

(1)

We observe that there is a strong resemblance between this expression for the one dimensional and two dimensional two-channel filter bank. Particularly, the above filter bank provides a perfect reconstruction i.e. $\tilde{X}(w) = X(w)$ if and only if it satisfies the following conditions.
\[ H_0(w)F_0(w) + H_1(w)F_1(w) = 2 \]  
\[ H_0(w + \pi)F_0(w) + H_1(w + \pi)F_1(w) = 0 \]

When the filters are restricted to be finite impulse response (FIR), the perfect reconstruction conditions imply that the synthesis filters are specified by the analysis filters

\[ F_0(z) = z^kH_1(-z) \]  
\[ F_1(z) = -z^kH_0(-z) \]

The filter bank can be used to split the frequency spectrum of the input signal into a lowpass and a highpass channel using a diamond-shaped filter pair. Frequency characteristics of these filters are shown in Figure 3.

\[ H_0(w_1, w_2) = H_0(w_1 + \pi, w_2) \]  
\[ H_1(w_1, w_2) = H_0(w_1, w_2 + \pi) \]

[3] Directional Filter Bank

a) First level of DFB
Construction of the first level of DFB requires two filters which are \( H_0(w_1, w_2) \) and \( H_1(w_1, w_2) \). They have hourglass-shaped like passbands as shown in Fig. 3b. Hourglass-shaped filters are created by applying the modulator \( e^{-jw_1\pi} \) to diamond shaped filters \( H_0(w, w) \) [12]. So the relation between Diamond-shaped filters and Hourglass-shaped filters used during the first level of DFB can be given as

\[ H_0(w_1, w_2) = H_0(w_1 + \pi, w_2) \]  
\[ H_1(w_1, w_2) = H_0(w_1, w_2 + \pi) \]
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Figure 4: (a) Three level DFB. Output of First level is input for second level and so on

b) Second level of DFB
Subbands produced by the first level are used as input to the second level as shown in Fig.4. The filters required for the construction of the second level are $H_0(Q^T(w_1, w_2))$ and $H_1(Q^T(w_1, w_2))$ where $T$ represents transpose. Filters of this level of DFB are given by following equations:

\[
H_0(Q^T(w_1, w_2)) = H_0(Q^T(w_1 + \pi, w_2)) \quad (8)
\]
\[
H_1(Q^T(w_1, w_2)) = H_0(Q^T(w_1, w_2 + \pi)) \quad (9)
\]

c) Third level of DFB
Subbands produced by the second level are used as input to the third level. Filters used during the third level of DFB are $H_0^i(w_1, w_2)$ and $H_1^i(w_1, w_2)$ where $i = 1, 2, 3, \text{ and } 4$. Overall eight different filters are created to be used during this stage. Filters of this level of DFB are given by following equations:

\[
H_0^i(w_1, w_2) = H_0(R_i^T Q^T Q^T(w_1 + \pi, w_2)) \quad (10)
\]
\[
H_1^i(w_1, w_2) = H_0(R_i^T Q^T Q^T(w_1, w_2 + \pi)) \quad (11)
\]

Where $R_i$ are resampling matrices given as follows

\[
R_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad R_2 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad R_3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad R_4 = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}
\]

[4]Vessel Model
With assumption that in images, vessels are bright over the dark background and the brightness is decreased from their centers toward their boundaries. Therefore, a vessel is modeled as a tube with a Gaussian profile across its axis, which is identical to the x-axis [3].

\[ I(x,y) = \frac{c}{2\pi\sigma^2} \frac{y^2}{\sigma^2} \]  \hspace{1cm} (12)

The Hessian of model can be expressed as

\[ H = \begin{bmatrix} \frac{\partial^2 I}{\partial x^2} & \frac{\partial^2 I}{\partial x \partial y} \\ \frac{\partial^2 I}{\partial x \partial y} & \frac{\partial^2 I}{\partial y^2} \end{bmatrix} \]  \hspace{1cm} (13)

And its Eigenvalues are

\[ \lambda_1 = 0; \quad \lambda_2 = \frac{y^2 - \sigma^2}{\sigma^4} I \]  \hspace{1cm} (14)

In Hessian-based vessel enhancement approaches, a vessel is declared when the ratio between the minimum and maximum Hessian eigenvalues is low. Its direction is considered to be the eigenvector corresponding to the smallest eigenvalue in absolute value. The Hessian of the above model can be expressed as above. [3] In order to capture vessels with various sizes, one should compute the gradient and the Hessian at multiple scales \( r \) in a certain range. In this case, the only way to ensure the well-posed properties, such as linearity, translation invariance, rotation invariance, and re-scaling invariance, is the use of linear scale space theory, in which differentiation is calculated by a convolution with derivatives of a Gaussian.

\[ I_x = \sigma^\gamma G_{x,\sigma} \ast I; \quad I_y = \sigma^\gamma G_{y,\sigma} \ast I \]  \hspace{1cm} (15)

where \( I_x \) and \( I_y \) are, respectively, the spatial derivatives in x- and y-direction of the image \( I(x,y) \) and \( G_{x,\sigma} \) and \( G_{y,\sigma} \) spatial derivatives of a Gaussian with standard deviation \( \gamma \). When applying the multiscale analysis, the model in[4] is convolved with a Gaussian of standard deviation \( \sigma \). The parameter \( \gamma \) is used [5] to normalize the derivatives of the image.

\[ G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \]  \hspace{1cm} (16)

This normalization is necessary for comparison of the response of differentiations at multiple scales because the intensity and its derivatives are decreasing functions of scale. In vessel enhancement application, where no scale is preferred, \( \gamma \) is usually set to one.


In order to compute the Hessian eigenvalues with less noise sensitiveness, it is necessary to align the vessel direction with the x-axis. One possible way is to rotate the directional images. As image rotation means interpolation of images, interpolation may cause artifacts and thus is harmful especially in case of medical images. We therefore rotate the coordinates instead of the directional images. Suppose the directional image \( I_i \) (\( i = 1, \ldots , l \)) corresponds to the orientations ranging from \( \theta_i,\min \) to \( \theta_i,\max \) (counterclockwise angle). Its associated coordinates \( O_{xy} \) will be rotated to \( O_{x'y'} \) by an amount as large as the mean value \( \theta_i \).[11]
\[ \theta_i = \frac{\theta_i, \text{min} + \theta_i, \text{max}}{2} \]  

(17)

Then Hessian matrix of the directional image \( I_i \), in the new co-ordinate is determined as

\[
H' = \begin{bmatrix}
\frac{\partial^2 I}{\partial x^2} & \frac{\partial^2 I}{\partial x \partial y} \\
\frac{\partial^2 I}{\partial x \partial y} & \frac{\partial^2 I}{\partial y^2}
\end{bmatrix}
\]  

(18)

Where

\[
\frac{\partial^2 I}{\partial x^2} = \frac{\partial^2 I}{\partial x^2} \cos^2 \theta + \frac{\partial^2 I}{\partial x \partial y} \sin 2\theta + \frac{\partial^2 I}{\partial y^2} \sin^2 \theta
\]  

(19)

\[
\frac{\partial^2 I}{\partial y^2} = \frac{\partial^2 I}{\partial x^2} \sin^2 \theta - \frac{\partial^2 I}{\partial x \partial y} \sin 2\theta + \frac{\partial^2 I}{\partial y^2} \cos^2 \theta
\]  

(20)

\[
\frac{\partial^2 I}{\partial x \partial y} = -\frac{1}{2} \frac{\partial^2 I}{\partial x^2} \sin 2\theta + \frac{\partial^2 I}{\partial x \partial y} \cos 2\theta + \frac{1}{2} \frac{\partial^2 I}{\partial y^2} \sin 2\theta
\]  

(21)

The Hessian eigenvalues are then defined by the diagonal values of \( H' \)

\[ h_{11} = 0; \quad h_{22} = \frac{y^2 - (\sigma_0^2 + \sigma^2)}{(\sigma_0^2 + \sigma^2)^2} I(x,y) \]  

(22)

Where \( \sigma \) selected in a range \( S \) is the standard deviation of the Gaussian kernel used in the multiscale analysis. Practically, the vessel axis is not identical to the \( x' \)-axis and hence \( h_{11} \approx 0 \). Inside the vessel \( |y'| = \sqrt{\sigma_0^2 + \sigma^2} \) and thus \( h_{22} \) is negative. Therefore, vessel pixels are declared when \( h_{22} < 0 \) and \( \left| \frac{h_{11}}{h_{22}} \right| \ll 1 \).[4]

[6] Recombination of directional images

The final output image can be obtained by simply summing all directional images or by using weight scale for each directional image based on vessel similarity measures mentioned by F. Zhou et al.[1]. Finally all images are recombined to generate final result i.e. enhanced image.

DFB has excellent application in various image processing applications. In this paper we have reviewed an application of Directional filter bank in medical image processing for the reduction of noise from images for detection of the blood vessels.

REFERENCES:


